

## AEROELASTICITY IN PRACTICE

B. O. HEATH

Project Manager  
English Electric Aviation, Ltd.

### ABSTRACT

The paper reviews some changes of emphasis which have taken place in the application of aeroelastic theory to high speed aircraft over the last fifteen years.

Aircraft such as the Canberra (Fig. 1) had geometric characteristics for which flutter calculations based on structural beam and aerodynamic strip theory should have been sufficient, but complications are described which arose in practice.

The shape of aircraft, such as the Lightning (Fig. 2), designed for higher speeds at supersonic Mach numbers made the application of such concepts dubious—even for initial calculations. For complete clearance, loading and static aeroelastic calculations as well as flutter had to use surface theory: the greater demands of subsequent aircraft need the use of these theories from the start.

It is apparent that the problems of aeroelasticity permit no watertight compartments to exist in a design organization; close coordination and feedback of experience between designers, aerodynamicists, stress, servo mechanisms and test engineers are essential at all stages. For example, evaluation of derivatives by flutter engineers must not be divorced from the quasi-static values derived by the aerodynamicist.

Despite improved methods of calculation, estimation of stiffness has continued to be difficult and will demand further care. It will be appreciated that large computations based on matrix methods now form an integral part of this process; they enable a realistic structure to be examined theoretically rather than an idealized representation amenable to more traditional methods. Engineers can be released for interpretative work and for improving detail design which must finally make manifest the advances in overall accuracy.

There are two sections: the first deals with oscillations, the second with loads and static aeroelasticity.

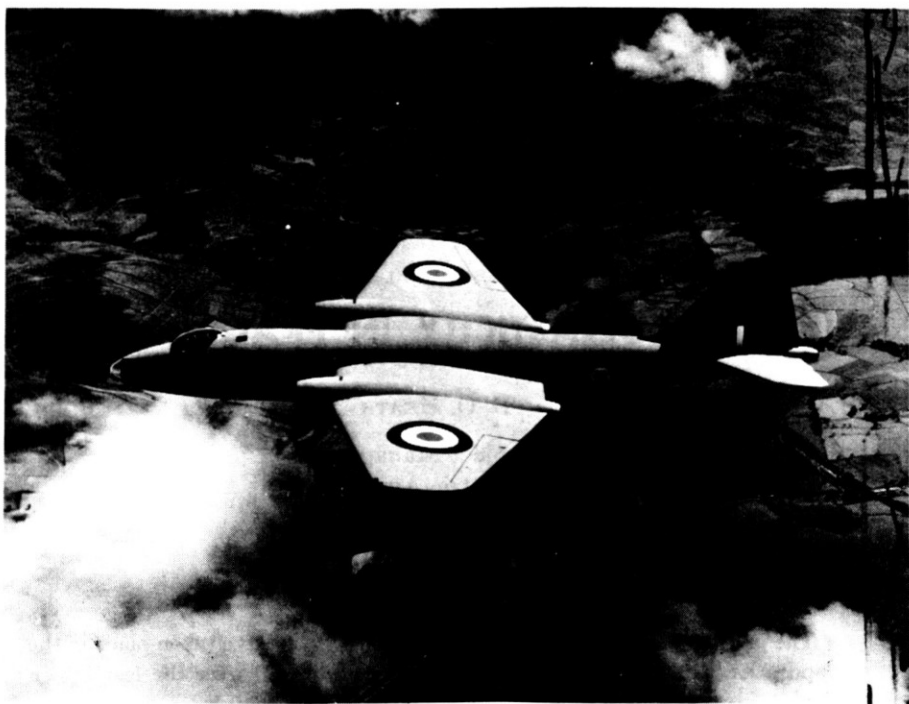


Fig. 1.

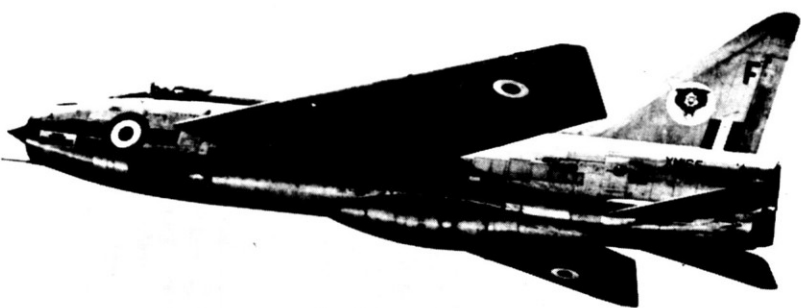


Fig. 2.

## INTRODUCTION

This review of some aeroelastic experience does not claim to present new concepts but is intended to illustrate how separate theories and facilities were integrated at one aircraft firm.

No representation of the pressure of timescale is attempted but development has been sufficiently rapid for one type to climb faster than its original design diving speed. Consequently, a procedure which enables safety to be maintained while its unknowns are progressively removed is far preferable to an elegant closed solution which cannot be applied in time.

### PART 1. DYNAMIC AEROELASTICITY

#### MODE SHAPES AND FREQUENCIES

##### UNSWEPT SURFACES

At the start of the period under review, structural aspect ratios were sufficiently high for flexural axes and simple modal shapes in torsion and flexure from encastered roots to be assumed for initial flutter calculations, but adjustment was necessary when resonance tests were completed. Recalculation was necessary when inclusion of body freedoms and differences in end slope caused the effective node to approach tail-surface mass balance more closely than with the fixed root and arbitrary modes initially assumed. The necessity to account for such effects led to the calculation of free-free modal shapes from the best available stiffness and mass distributions. Desk machines at first formed the main assistance to computation and although methods due to Holzer, Stodola, and Myklestad were simple to apply for primary modes, the necessity to sweep these out of the iterations for the higher ones was tedious. Matrix methods proved simpler for computers to apply, leaving engineers free for interpretive work, and phased naturally into the programming of large digital computers when these became available. Provision of analog computers enabled a further check: it was easy to vary the frequencies of individual branch type modes on the simulator (initially set up for zero air speed and possibly applying a forcing sinusoidal) to obtain a better representation of tests on the ground before trying to reproduce those in flight.

The need for such adjustment was most apparent where nonlinear stiffness characteristics were relevant over low amplitudes of buffeting or under the comparatively small excitation which can be applied in resonance tests, and had to be reproduced in linear form. In the series of resonance tests on several production Canberra aircraft the modes not only varied from aircraft to aircraft but also with the level of excitation applied, to the extent of complete phase change between fin and tailplane bending in the primary antisymmetric mode. This particular case was due to production tolerances in the all-moving surface attachments, but such adjustments are equally applicable when extra stiffness is provided by secondary structure or where degradations due to buckling are involved.

Rapid changes of cross section seriously affected the basic stiffness data (even though diffusion theory had been applied in strength calculations), and led to drastic changes in mode shape, frequency, and to a missed mode on resonance tests although it was very apparent in flight. As an aid to diagnosis, results from strain-gaged resonance tests were employed; from an accurately defined experimental mode shape, frequency, and mass distribution, the bending moment in the resonance mode was obtained by double integration through shear force. Application of engineers' theory of bending then directly provided a distribution of stiffness. This was lower than estimated at the end of the armament bay where an access panel also led to reductions of lateral stiffness which were accentuated by a transport joint in the same region. In contrast, further forward the wing constraint in its chord plane gave a contribution to lateral stiffness so large as to amount to fixity. This enabled stiffening to be optimized by the attachment of an external doubler plate; flight tests confirmed the adequacy of the changes (Fig. 3). The Canberra tailplane flexure frequency was lowered from 15 to 12 cps by backlash, but on the Lightning such effects have been avoided by mounting the tailplane on stiff spigots with roller bearings.

#### SWEPT SURFACES

With the introduction of low-aspect-ratio surfaces of high sweepback on the Lightning, it became essential to use surface theory for stressing and this extrapolated to overall aircraft modes: not only was a flexural axis untenable but the accuracy of elastic stiffnesses based on engineers' theory was considered unreliable, particularly at the root trailing edge. The first method attempted for

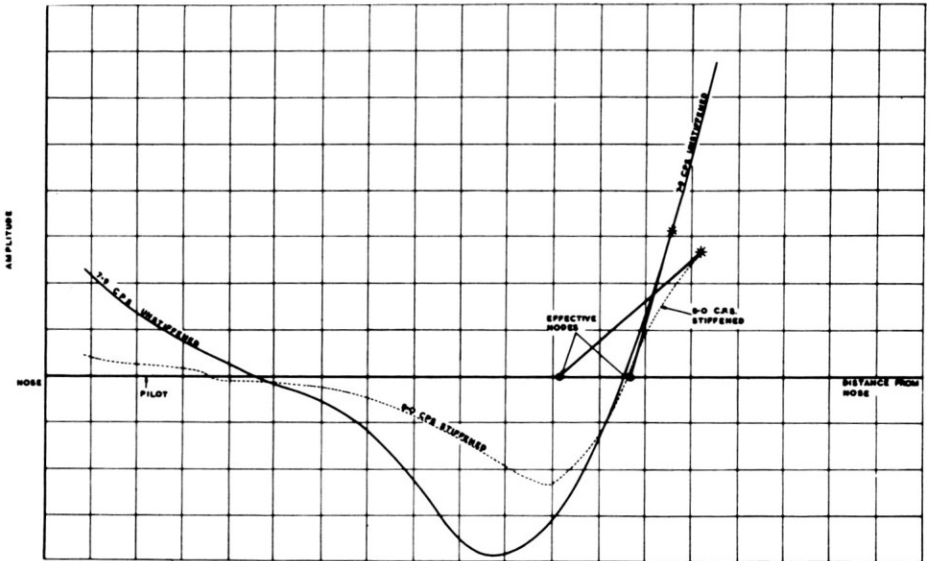


Fig. 3.

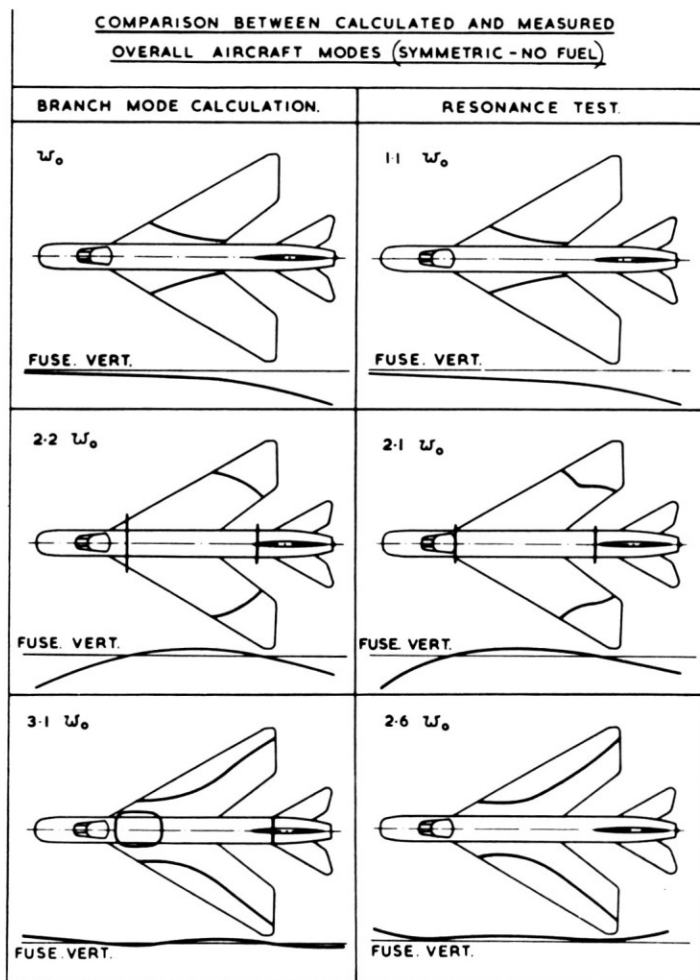


Fig. 4.

strength calculations was the manual relaxation of an equivalent load network based on spar and rib positions, but this proved too laborious by its slow convergence and was eventually replaced by matrix manipulation of deflections with elements of stiffness based on the individual boom and panel sizes, balance of forces providing the necessary condition for the solution (and occasionally vice versa—i.e., compatibility of displacements on bodies). This procedure had the useful by-product of influence coefficients for use in static aeroelastic calculations and for modal estimation, with masses lumped to the node points.

Another method is to synthesize overall modes by Lagrangian methods from component fixed root branch modes and body freedoms, the individual branch modes themselves being provided by matrix calculations on the encastered wing, fuselage, tail, etc. When compared with full-scale resonance tests (Fig. 4) good agreement on the first three of four modes was obtained by either method, but

those higher modes involving branch overtones were not satisfactory: test modes could not always be reproduced even from branch modes adjusted after resonance tests. There were several possible reasons for this structural damping of the fuselage was demonstrably high: tail modes could not be easily excited by wing excitation or vice versa, either on test or on the simulator, and this, combined with close frequency coincidence of wing and body torsion, wing overtone bending and tailplane bending was believed largely responsible for the difficulty. Fortunately, the fuselage modes are little affected by airspeed and the flexibility of the branch method has enabled flight coverage to be given. (In this connection, a direct solution for flutter speed and mode by including complex aerodynamic terms with the structural influence coefficients has been proposed for use where flight modes are believed to be far removed from any zero speed modes in combination, and has been found practical in application) (Fig. 5).

On the all-moving tailplane surfaces, branch modes (involving tailplane torsion rotation, for example) were combined to give a (greater) number of derived modes. These contained modes of very much higher frequency, specifically five times the fundamental. Omission of such modes in subsequent flutter calculations has been shown to be unimportant, as would be expected intuitively.

When there is no significant difference in mode shape and frequency, it has been found simpler to use the theoretical values as being truly orthogonal. Where only nominally normal experimental modes have to be used, the cross products of inertia have been set to zero in the flutter equations pending better tests or recomputed modes. Such refined resonance tests have involved the suppression of unwanted modes by judicious ballasting.

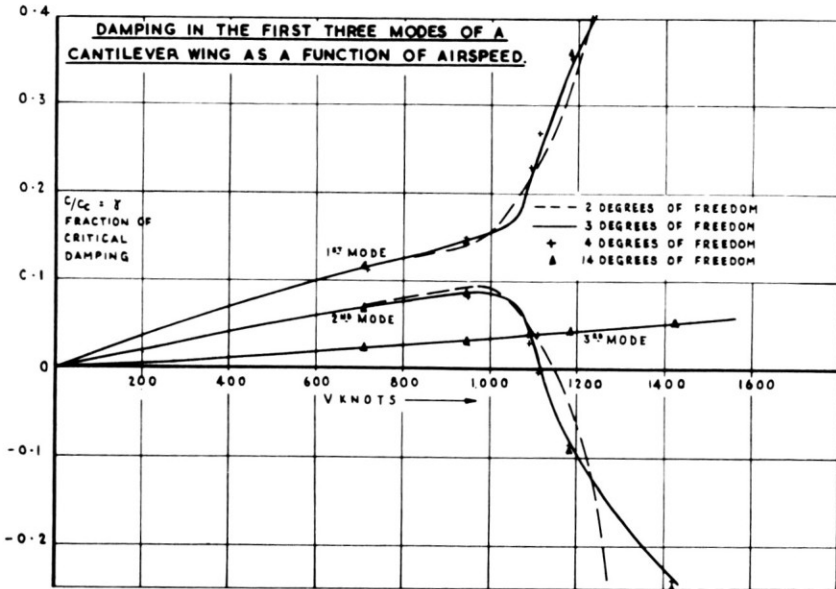


Fig. 5.

## DERIVATIVES AND BALANCING

Derivatives based on two-dimensional potential flow theory had given a reasonably adequate basis for main surface aerodynamic forces and for control surface calculations which had provided mass-balancing criteria. Such theory was difficult to relate to particular control surfaces with balancing by horn or sealed beak, and became grossly misleading on control surfaces and tabs—even if of simple form—when these were affected by shock waves and associated boundary-layer separation. It was possible to waste considerable manpower in evaluating second-order frequency effects on theoretical derivatives which were incorrect to first order and often of the wrong sign compared to the quasi-static values known from “aerodynamic” sources, i.e., derived with reference to empirical, wind-tunnel, and flight-test results. The flutter notation differed from that of aerodynamics and this did little to help resolve the difficulty. Because of this emphasis, Aerodynamics took responsibility for flutter calculations and with more realistic derivatives the flight frequencies and airspeeds were reproduced by calculation. Canberra flight tests using quasi-static control applications were used to determine the values of significant rudder derivatives and although the degrees of freedom provided by the spring tab did not help obtain precise answers, they were sufficient to confirm bounds of probability for the rudder direct and aerodynamic cross stiffnesses.

In the absence at that time of a large available high-speed tunnel, full-scale low-speed tests on a section of wing and aileron were made which, despite the low speed, were of considerable use in providing an overall datum for the stiffness derivatives. Detailed information on the effectiveness of the beak seal by pressure plotting was provided by pressure plotting (and permitted rolling cases to be more rigorously covered).

Briefly, for the Canberra tail surfaces at frequencies under 8 cps, adjustment of the direct aerodynamic control surface stiffness to avoid coincidence with main component frequencies was effective, but represented a compromise with stability and control considerations as is so typical: fortunately, provision for this had been made in the basic design of the tail surfaces where easy adjustment of the horn balance was available and no final embarrassment ensued.

Removal of aerodynamic cross coupling was similarly effective, and forward mass balance proved beneficial throughout in line with theory.

After a report of 24 cps oscillations on an aircraft assigned to experimental flying, calculations showed the importance of tab direct mass inertia, tab product of inertia about the control and tab-hinge lines, and the tab direct aerodynamic stiffness. The variation in damping noted by the pilots in flight clearance of production aircraft had given some correlation with variable mass characteristics attainable within production tolerances: this clue was exploited by adding tab-mass balance for flight tests in such a manner that the mass variations quoted traversed several computed flutter bounds at positions depending upon the tab direct aerodynamic stiffness assumed.

Demarcation defined in flight between satisfactory and unsatisfactory damping fixed the aerodynamic tab stiffness value which must have applied, and this was then used in further calculations. To remove second-order effects,

routine balancing of main surfaces to closer limits was introduced; production flight tests to raised levels confirmed that the cure was complete.

Higher-frequency vibrations on Canberra were in general improved by lighter tab surfaces which gave lower direct inertia and increased the favorable product of inertia about the hinge lines. Taut skinning avoided increase of trailing-edge angle under flight suction, which had led to undesirable loss of direct aerodynamic tab stiffness.

On the Lightning, the power controls did not entirely remove the need for mass balance; failure cases had to be covered, but even in the controls working cases, mass balance proved economic in eliminating flutter at the design stage, e.g., on the aileron, without appealing to excessively high impedances for which weight penalties would have occurred, and which could have intensified the task of the control designer in the light of knowledge then existing.

**COMPARISON OF STEADY LOADING BETWEEN THEORIES FOR TWO ARBITRARY MODES**

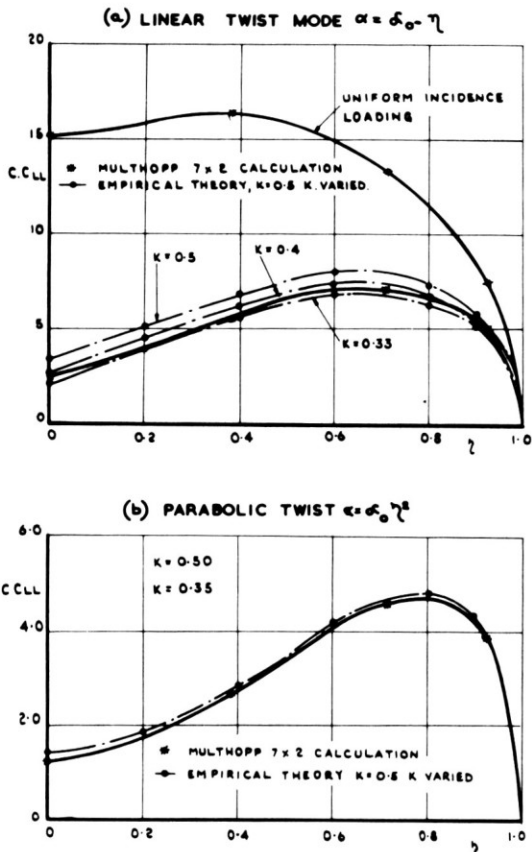


Fig. 6.



TAILPLANE NORMAL MODES COMPARISON  
OF  $l\alpha$  PART OF DAMPING LIFT DISTRIBUTIONS.

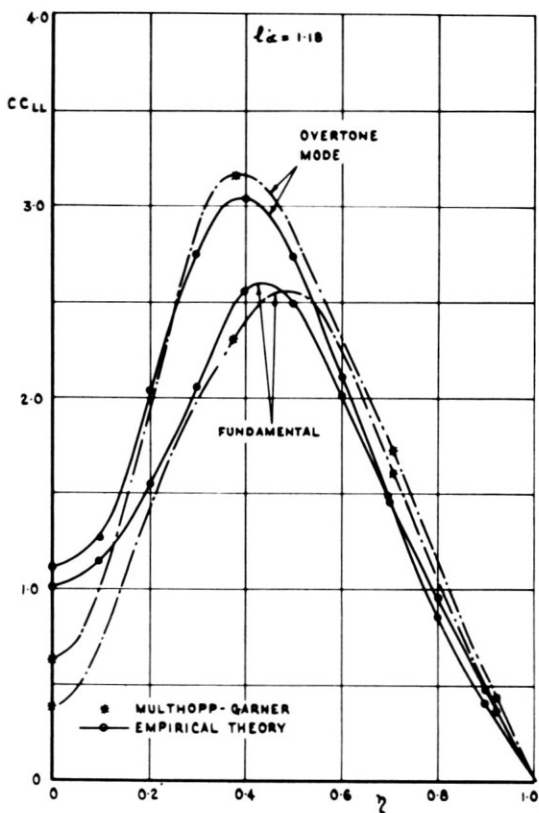


Fig. 7.

Although formal lifting surface calculations became available at a later stage, interim Lightning calculations were based on what may be described as a dynamic analogue of Schrenk's method for static spanwise loading. Examination of the loadings derived for static aeroelasticity (Fig. 6) and available dynamic surface loading (Fig. 7) data had shown that spanwise loadings could be synthesized from two components—the uniform incidence load weighted by the mode shape, plus a fraction of the uniform incidence mode decided by the spanwise integral of the first component—the latter term being a measure of spanwise induced effects.

#### EFFECT OF POWER CONTROLS

The representation of power controls in the flutter equations as an impedance having stiffness and damping as its components has been found satisfactory. Flutter speeds can be presented as a function of these components where they

do not vary appreciably with frequency, or where the damping component is completely overshadowed by the direct stiffness of the jack. This condition applies to the Lightning tailplane where a screw jack drive is used and, being generally conservative, was used in all early calculations. Where such simplifying conditions do not apply, impedance matching may be used: with stiffness and damping as coordinates, curves of constant flutter speed are plotted with the appropriate frequency noted or mapped thereon. By superimposing the variation of measured (or estimated) power control stiffness vector, again with frequency noted, it is possible to establish (Fig. 8) any coincidence or near coincidence of impedance with that which would give flutter at the same frequency. To date, however, the damping component has not been found important.

The estimation of power control impedance provides material for a paper in its own right: impedance at critical flutter frequencies must be adequate to avoid flutter, and a high static stiffness (or zero frequency impedance) is required in order that aerodynamic forces do not drastically reduce the control angle for a given input. In design calculations for impedance, the representation of oil compressibility and jack attachments as equivalent springs with terms for structural and position feedback in the equations of motion gives results which

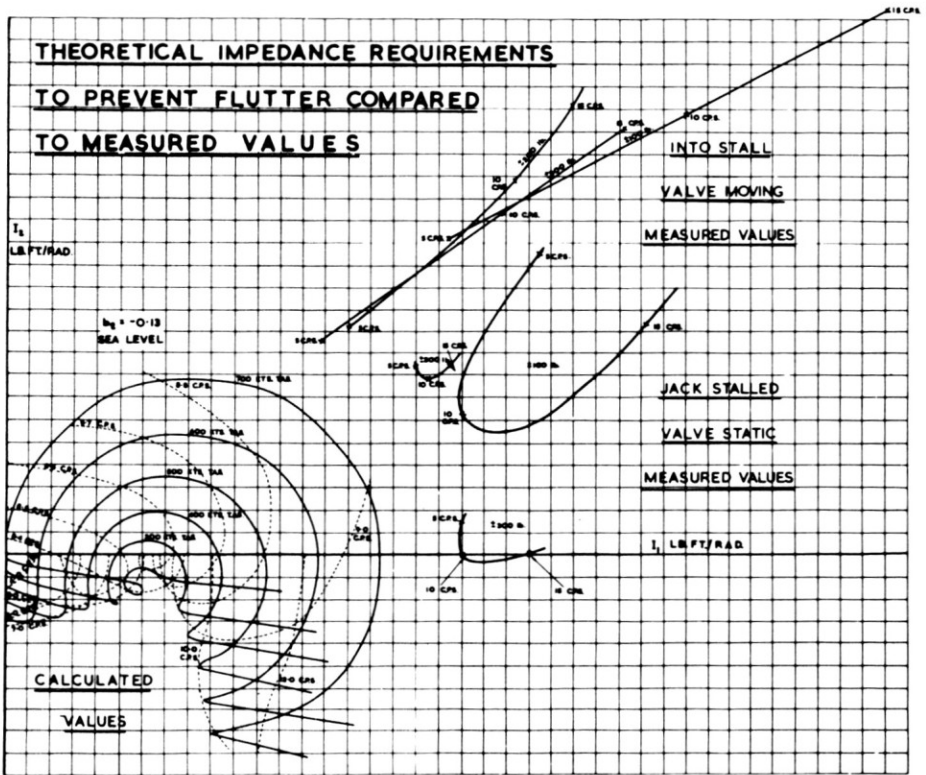


Fig. 8.

compare favorably with the results of more comprehensive analog simulation. To cover high-temperature cases, the bulk modulus for the oil at its maximum operating temperature is considered.

Failure cases also need to be covered—for example, on a tandem or dual jacks, the failure of one pressure supply. This had little effect on the Lightning aileron which has two piston jacks with coupled inputs.

The minimum value of zero frequency impedance likely to be encountered is when the jack is stalled by high thrusts acting on it. The control valve is then wide open so that position feedback has no further significance; oil compressibility in the system and attachment stiffness decide the impedance.

To determine the impedance by test a sinusoidally varying load is applied to the output (its magnitude is known from strain gages on the jack body previously calibrated in a test machine) and the phase and amplitude of the response monitored over the relevant frequency range. Test results usually have considerable scatter but generally show the analytical predictions to be adequate. As an aircraft is not usually available as soon as its power controls, tests are carried out initially in a rig and corrected for structural stiffness. It has been usual to test with the input locked so that the valve lies within its overlap to give an end point static value, with various rates of input (as in practice), and with the unit going into stall or stalled. Rate of loading does not greatly modify the stiffness once friction effects are passed, but phase changes are induced, and variation between direction of stroke has been apparent. At lower frequencies and higher force levels, phase lead has been recorded, although lag is usual.

Changes in overlap have caused little effect and tests after endurance cycling have exhibited only small change in characteristics, mostly on the phase differences.

Overall stability and integrity are checked as part of the general aircraft clearance. This does not fall within the scope of this paper, except to mention that careful mass balancing of the input circuit has proved necessary in rigid body modes, and that the most practical method of clearing that circuit dynamically is by controlled jerk tests (using springs to provide the impulse) and resonance tests, both with the circuits live. This shows up any instabilities and frequency coincidences, but does not completely cover the effects of aerodynamics.

Duty cycles set up for proving power control integrity included the effects of demonstration flying, and with the growing popularity of formation displays, the effects of adjacent aircraft may have to be assessed for the loading cycle.

When electronic aids to aerodynamic damping or stiffening are installed care must be taken to insure that the pickoffs associated with these are not sited on structural antinodes and that the gain provided is not high enough to provide an unstable loop through the power controls and autostabilizer system at structural frequencies.

As the Lightning does not require such devices to provide satisfactory basic characteristics no difficulties have persisted, but oscillation did occur during early flying with roll rate gyros too near antinodes, and with couplings not revealed by the simulation of the ground rigs.

## MODEL TECHNIQUES

To provide a check on the flutter characteristics of the Lightning wing, so that skin gages could be confirmed pending the conclusion of preliminary calculations, three 1/7th scale wings were provisioned for free-flight rocket firing. To respect similarity, thin light alloy surfaces had to be used and stabilized against buckling; a balsa filling was selected. Unfortunately, this led to considerable unwanted stiffness, which persisted despite extensive slotting on the model. (When tested, the aircraft wing itself proved stiffer over low loadings than calculated, due to secondary structure picking up load through fastener friction, etc., although at proof and ultimate the theoretical load distribution was in good agreement.)

The first and second rocket models showed no divergent oscillations up to and satisfactorily beyond the design diving Mach number of the aircraft, but had phases of continuous vibrations at the wing fundamental bending frequency, reminiscent of buffeting. This could have been induced by the configuration of the rocket boosters, so a cleaner third model was built, which was used to check roll rate and aileron reversal at the same time. Again, no divergent oscillations occurred, but the vibration persisted at a lower amplitude. As buffeting presents no difficulty on the Lightning—such as does exist is believed to emanate from the fin—rough air may have been the cause in the rocket model. No roll reversal occurred (due to the location of the aileron on the main spar extremities there is little twist), although a reduction of effectiveness due to bending did occur, in good agreement with theory (Fig. 9).

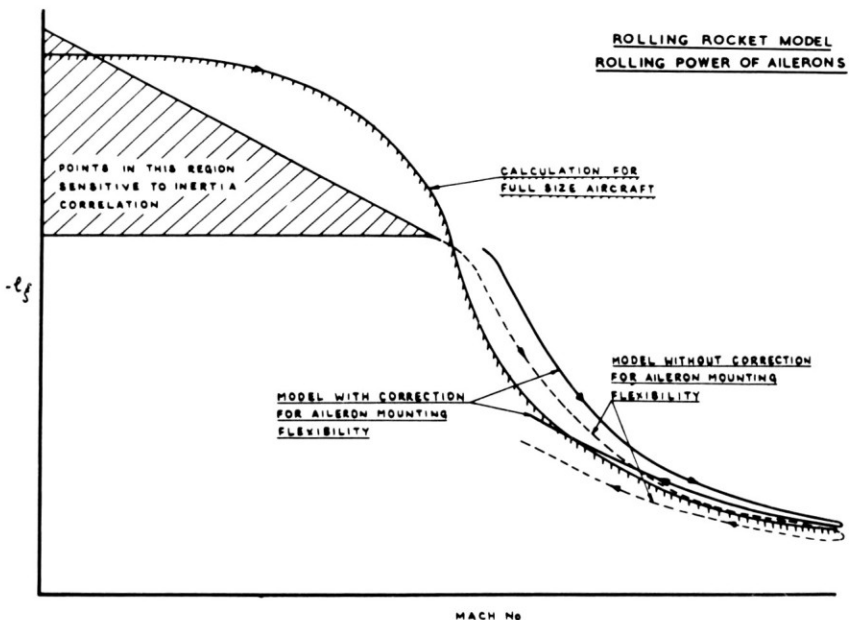


Fig. 9.

The setting of the Lightning tailplane spigot out of the tailplane chord plane, to give acceptable fairing on the fuselage, gave some concern as to dynamic couplings which might occur—such as fore and aft bending with rotation. To explore these, a quickly constructed device capable of easy adjustment was required. Cover over the whole Mach number range was not believed essential, as any unusual effects of the geometry would doubtless be reproducible at low speed, given sufficiently adaptable stiffnesses. A Xylonite model was soon proposed, but was subject to the criticism that no such model known at that time had fluttered—allegedly due to the high structural damping of Xylonite or the variation of its Young's modulus with frequency. To resolve these particular issues a segmented model unswept wing was constructed with a single spar. When subjected to resonance tests, fore and aft bending occurred and obscured the normal bending and torsion modes—even though the configuration was completely conventional. The mass of the exciter was sufficiently great in comparison to the model to give difficulty, and finally a manual displacement was found to give satisfactory vibrations of which the wave forms were recorded optically. From their decay a structural damping coefficient of 0.08 was deduced, confirming part of the objection.

The shear and Young's moduli were determined over a range of frequencies and were found to increase from the static value as suspected. When this effect was taken into account, good frequency agreement was obtained; the modal shapes were in good agreement with calculations throughout. The flutter speed with an added movable mass followed the variation calculated, but was higher in actual speed than theory throughout.

A representative half-scale Xylonite model of the tailplane was constructed for testing in the Warton low-speed tunnel, with a glass cloth spigot to reproduce the correct ratio of stiffnesses for light alloy to steel. To preserve scaling parameters, extensive lead shot loading was employed, the skin thickness then following the speed scaling. When tested, supported from the wind tunnel roof, the stiffness was lowered by leading-edge skin buckling, which raised the flexure/torsion frequency ratio, and no flutter was reproduced as a result. However, damping was low at the highest tunnel speed and in reasonable agreement with flutter calculations; no unusual couplings were observed and the aircraft was cleared for flight with greater confidence on the basis of calculations using conventional modes. (Later calculations showed fore and aft bending could be stabilizing.)

Single degree of freedom flutter, as represented by a loss of direct damping in surface rotation, was explored by a  $\frac{1}{16}$ th scale wind-tunnel model of the Lightning fin and rudder in the Warton transonic tunnel. The rudder was of Tufnol and had a steel spar. It was mounted on to a dural fin by steel bars of variable stiffness so that the frequency parameter could be changed. Negative damping was found between Mach numbers of 0.95 and 1.2, agreeing fairly well with estimates.

In flight, the transonic aft movement of the rudder-induced sideload was found more important, producing couplings with the fin bending, and a reduction of rudder span was effective in curing this. The horn mass balance was removed

in the process, but this gave a much higher frequency to the torsional rudder mode, which enabled the stiffness of the power control to be effective both in that mode and with fin bending subsonically. Manual reversion was deleted by using tandem jacks.

## FLIGHT TEST TECHNIQUES

### “BONKERS”

At low frequencies, stick jerking proved valuable on Canberra, but due to the relatively low cut off characteristics inherent in stable power controls, generally these have not proved a suitable medium (in our experience) for providing flight excitation for flutter tests. On a high-density modern aircraft without an armament bay, little volume exists for the installation of inertia exciters so that the success of small explosive charges has been welcomed. These appear to be quite adequate to cover all important modes if sited with reference to the best data on antinodes, and have proved economic in flight testing time. The range of instrumentation pickups and their positioning should similarly receive careful study.

### “ROUGH AIR EXCITATION”

On the Canberra the vibrations which were true flutter mostly occurred quite early in flight development of the prototypes and were, of necessity, quickly solved (see above) by conventional means, but not without considerable effort. Of more lasting concern were the limited amplitude oscillations which remained due to the low damping of the tail modes. These oscillations assumed a pure form at high indicated speed at the fundamental 6-8 cps frequency, as opposed to those at high subsonic Mach number where a more ragged record involving higher frequencies was obtained.

Such effects become most severe on production aircraft (requiring early release) in periods of good weather, but examination of Meteorological Office records of lapse rates and the application of Richardson's and Scorer's criteria for turbulence gave no significant correlation with flight difficulties and their day-to-day variation.

In order to rationalize reporting, to discover and separate the dependence of these oscillations on speed and Mach number, a systematic flight plan permuting six indicated air speeds and five Mach numbers was derived. Oscillograph records at several airframe stations were taken on each flight at the twenty-odd practical flight conditions which resulted. This was necessary as pilot opinion could not represent conditions over the whole airframe, and qualitative reports involving improvement and deterioration in different flight conditions were difficult to assess (even after the pilots' notation had been demonstrated to the engineers concerned by flight experience!) Traces had to be analyzed manually at that time, and this was not easy where overtone modes and ragged wave forms obscured the basic 8 cps traces whose amplitude and phase relationship were desired. After such analysis “a 3-D plot” was necessary (Fig. 10) to show the form of variation but still gave little assistance in diagnosis or assessment of

proposed cures. *Ad hoc* refinement of control and undercarriage gaps, mass balancing, tab-skin stiffening were stop gap cures at this stage.

Checks on the frequency of aerodynamic pressure variation by any known method showed these to be higher than those of the actual aircraft response, which was essentially at its fundamental frequency in an asymmetric mode which was of a very consistent shape. As a result, the application of sinusoidal forcing to a single lightly damped system at a frequency well above its fundamental was considered, and was related by similarity laws to a family of aircraft assuming that their stiffness followed the then-current standard British requirements for military aircraft. This showed that larger aircraft would experience the more severe amplitudes of buffeting—other conditions being equal, it was evident that for its size the Canberra was being flown far faster near sea level even than other aircraft of higher Mach number capability. The calculation showed that the variation of buffeting on any given aircraft should have reduced to (Mach number times a function of Mach number) when multiplied by the fraction (square root absolute temperature divided by relative pressure). This was found to be the case, with the function of Mach number similar to a drag rise, when the flight results were so processed. This enabled the results of modifications to be evaluated more precisely, and a solution to be reached (see Fig. 11 and following paragraphs).

#### SOURCE OF EXCITATION

To determine the origin of the rough air emanating from the forward sections of the Canberra which was exciting its tail surfaces, flow-visualization methods proved very useful. Initially, iodine solution was sprayed from likely sources so that it would stain the rear surfaces which were sprayed with starch, but the degree of staining was insufficient and dye was finally substituted. This showed

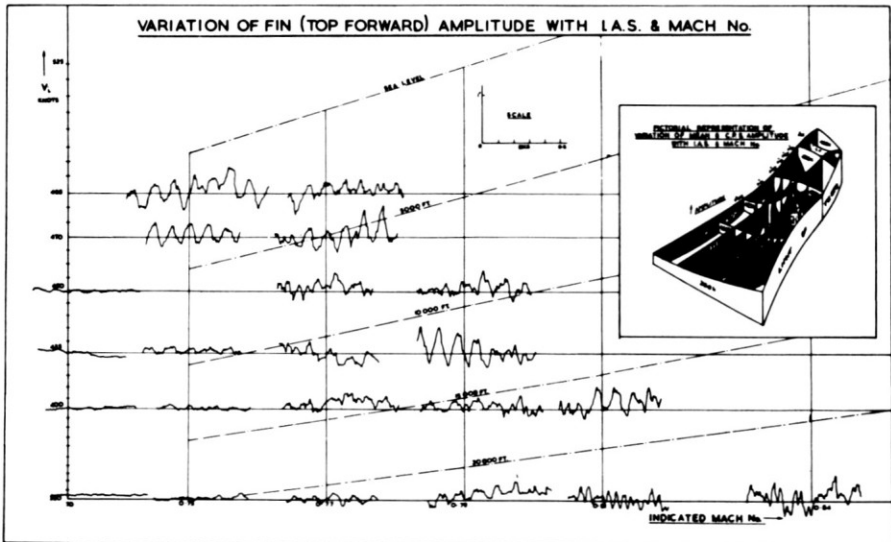


Fig. 10.

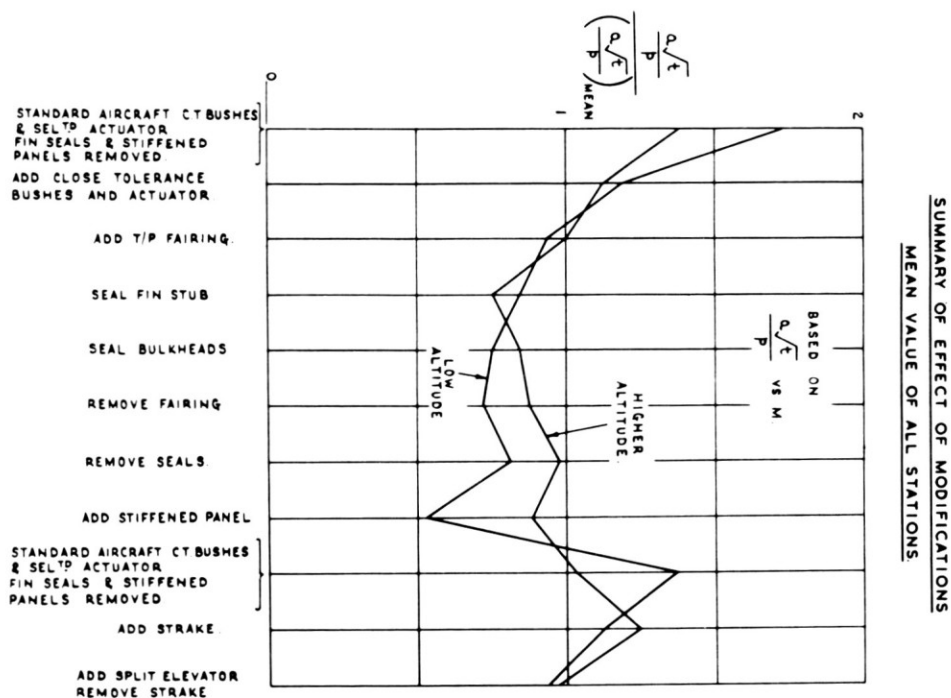


Fig. 11.

that the wakes from the canopy, the wing root, and the bomb doors all followed the upsweep of the rear fuselage to the elevators, but little could be done to change this. However, there was sufficient staining inside the fuselage where the tailplane center box passed through it to show that internal pressure differentials were also acting on the flow. Suspicions that the fin and rudder shroud gaps were opening under the actions of pressure differentials on the shroud had already led to the installation of pressure pickups in that region. These had detected a pressure variation changing in the same way with Mach number as the corrected amplitude already described. By sealing internal gaps some improvement resulted; external sealing strips at the tailplane root proved still more effective, but were limited by changes to longitudinal trim (which normally had a desirable nose-up change as the Mach placard was approached). There was an incidental improvement in snaking characteristics, due to greater effectiveness of the rudder aft of its hinge line in direct Mach and cross aerodynamic stiffness.



## VORTEX GENERATORS

Flow-visualization studies using tufts showed (by the wear of the tufts) that separation was occurring on the base of the fin and enabled vortex generators to be positioned. They proved effective on the fin and were used there on several production aircraft, but when applied to the tailplane they produced inadmissible trim and stability changes and were not pursued.

## RUDDER CONSTRAINT BY POWER CONTROL

Flutter calculations suggested that the 8 cps flutter speed would be raised by fitting a stiff power control so that greater damping in the buffet modes could be expected at the lower speeds of actual flight. When a piston-type power control was fitted as a trial installation, 8 cps buffet was eliminated as predicted, but a 15 cps oscillation involving chiefly rudder torsion with jack-support deflection became apparent. However, aerodynamic sealing was sufficiently effective to

**STRUCTURAL RESPONSE TO HIGH  
MACH NUMBER BUFFET**

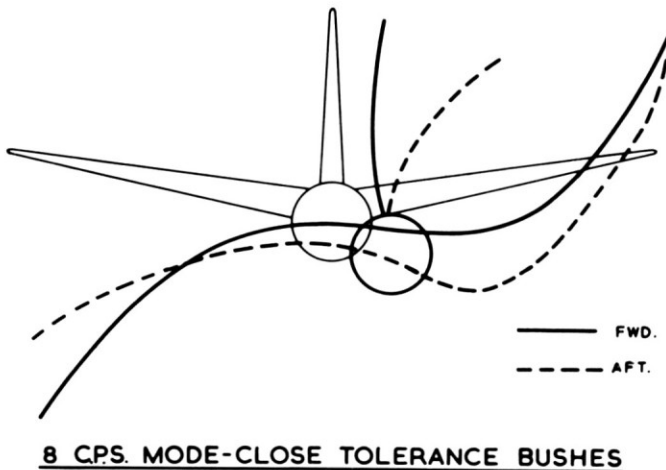
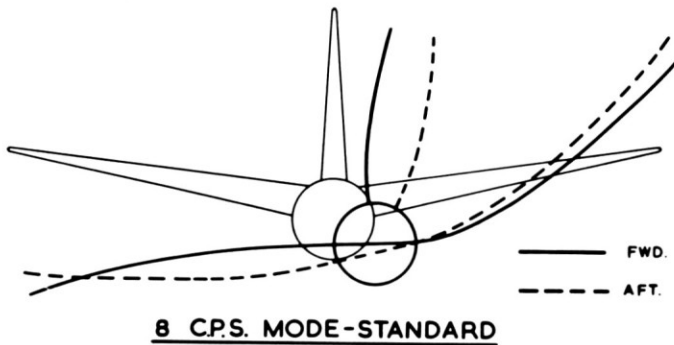


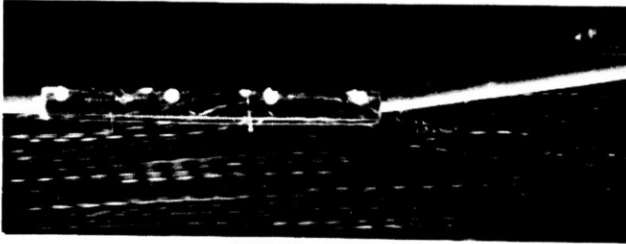
Fig. 12.

reduce the level of the latter to an acceptable value and the installation has subsequently proved satisfactory in service.

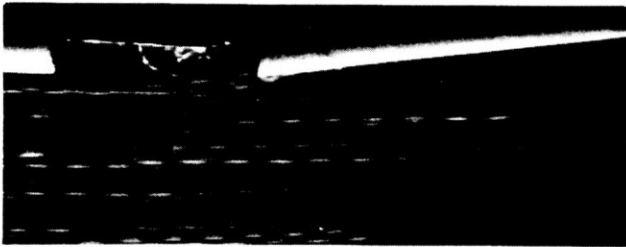
#### OTHER STIFFNESS CHANGES

Flutter calculations (using branch mode stiffnesses adjusted to approach measured primary modes and frequencies) indicated that increased tailplane bending stiffness, decreased fin bending stiffness and increased fuselage lateral bending stiffness would all be beneficial in raising the 8 cps oscillation limit speed. As described above, external stiffening plates were effective in raising the fuselage lateral frequency and making rudder-mass balance more effective so that the other less convenient methods were not necessary. Close tolerance bushes were tested on the tailplane bearings but gave such an extreme modification in mode shape that they were not finally fitted, despite generally favorable results, as the amount of new work implied was uneconomic bearing in mind the other improvements (Fig. 12). Similar factors weighed against the uncoupling of the elevator halves. This gave reduced 8 cps amplitudes by virtue of favorable

#### WATER TUNNEL FLOW VISUALISATION.



ORIGINAL BOMB BAY.



PR FLARE BAY.



ROTATING BOMB TRAY.

Fig. 13.

antisymmetric elevator-half rotations induced by their horn mass balances. A buffet in a 24 cps mode which normally involved tab rotation in coincidence with elevator torsion at high speeds was also eliminated, but frequency coincidence of a control rod with a lowered high-frequency bending mode of the elevator halves occurred—it could have been eliminated, but at the time was sufficient to terminate the development.

#### ARMAMENT BAY BUFFETING

Adaption of the Canberra to high-speed, low-altitude roles emphasized the importance of bomb-bay buffeting when a long bay is involved. Later flight experience on versions with a shorter armament bay installed showed these to have little increase in amplitude above the clean aircraft when their doors were opened. Such results were in agreement with water and wind-tunnel work, which showed length-to-depth ratio to be a critical parameter (Fig. 13).

If critical ratios of length/depth dimensions (about 6:1) cannot be avoided in basic design, a conventional bay may be improved by rounding the rear bulkhead, by fitting forward spoilers, by compartmenting the bays, or by augmenting the filling of the bay provided by the stores. Rotating doors can provide a more positive cure as demonstrated by the B-57 version of the Canberra.

As the frequency distribution of the excitation is a wide one, stiffening is only likely to raise the frequency at which buffeting occurs so that most cures rest in the field of aerodynamics.

## PART 2. LOADS AND STATIC AEROELASTICITY

### UNSWEPT WINGS

It was necessary to make aeroelastic corrections to the Canberra spanwise wing loading distribution to cover wing twisting which occurred at the higher end of its Mach number range, where the wing center of pressure moves forward (prior to its rearwards shift as  $M = 1$  is approached beyond the normal flight region). The nose-up wing moment was further increased on fitting tip tanks, whose forward body lift was increased by the upwash field still existing at the tip at the subsonic speeds involved.

These aerodynamic effects were magnified by the rearward position of the single main spar (forming the forward end of the torsion box) at 40 percent chord leading to an aft flexural axis. The skin gages were typical of light sheet-stringer construction designed to buckle about 30 percent ultimate, and it was necessary to correct for the postbuckling loss of stiffness. Search of literature made at the time showed a surprising lack of data on this and little evidence that it was considered. No doubt the increased sheet gages (or integral construction) of thin wings has now overtaken this shortage as far as the high-speed designer is concerned. The stiffness of the drop-tank mounting also contributed an important factor in deciding the divergence speed.

### SWEPT WINGS

#### INITIAL STAGES WITH SWEPT WINGS

Several lifting surface theories available for load calculations on the Lightning surfaces were found sufficiently adequate to agree with (or provide) overall stability derivative calculations, after thickness correction (Fig. 14). Vortex lattice theory due to Falkner was used, extended to  $M = 1$  by the application of coordinate adjustment to downwash function tables; linear theory was used at

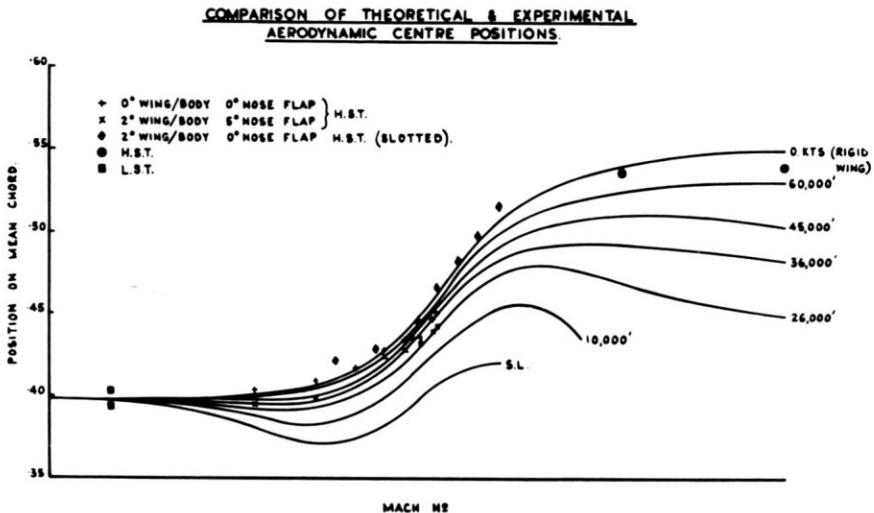
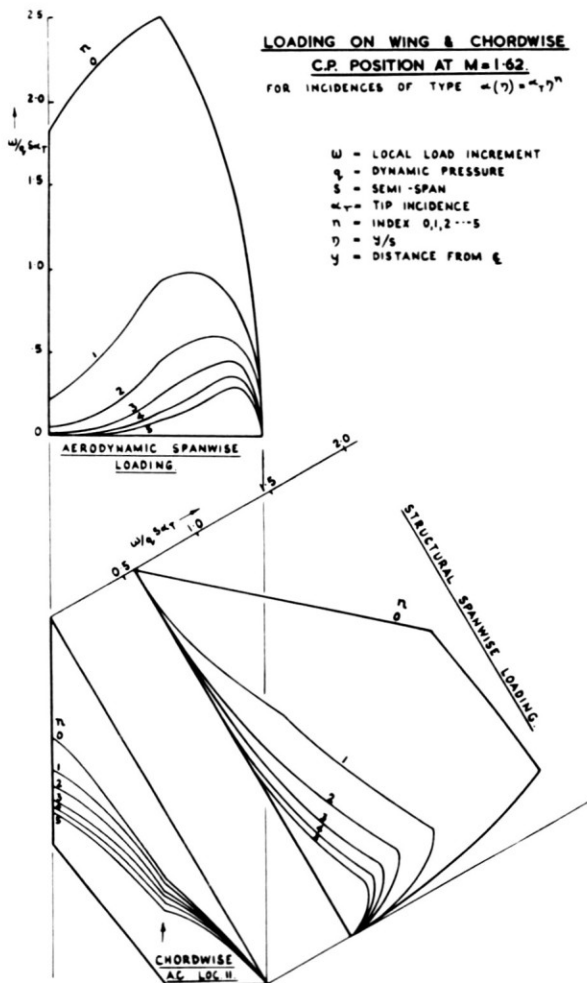


Fig. 14.



supersonic speeds. Aeroelastic distortions were covered by the assumption of power series distributions for normal deflection in terms of nondimensional structural span, with slopes resolved into aerodynamic incidences. Pressure distributions for these were resolved along the spars (Fig. 15) so that virtual work in increments of the modes assumed combined with strain-energy increments provided sets of linear simultaneous equations. Solutions of these were obtained at several speeds but ill-conditioning resulted at others. By combining the power modes so as to respect structural boundary conditions, this difficulty was overcome and one mode alone was finally found to predominate.

The tailplane treatment was similar, but as it is located close to the wing it could not be assumed to be in a uniform flow field. The downwash field from the wing was expressed as a power series of terms of nondimensional tailplane span to give the basic loading using the loads in the power series incidences.

On the all-moving tail surface, lack of continuous root constraint led to simple distribution modes—no more than linear twist being necessary—and solution was consequently quite simple. At a later stage, a refinement was made both to wing and tailplane by including the aeroelastic loading corresponding to mass-induced deflections.

To minimize hinge moments on the Lightning tailplane a swept hinge line is used, designed to pass through the range of center of pressures; any undue inaccuracy, while still apparently small in terms of chord, could be disastrous when expressed in terms of hinge moment. With the application of a modest tolerance, and an allowance for thickness, reliable values were obtained by the methods described. No difficulty has occurred during development and service, and wind-tunnel measurements before flight were also in good agreement.

In later work, finer networks were employed with more chordwise points, and the  $M = 1$  methods were refined.

#### LOADS, LATER STAGES OF SWEEPED WINGS

By evaluating the aerodynamic influence coefficients the pressure induced at one element can be related to the slope at another. On combining these influence coefficients with those of the structure, a solution by matrix manipulation or iteration on a digital computer becomes possible. This gives pressure differential distributions as a series of blocks which strictly require smoothing into the usual continuous representation (Fig. 16). Since the stressmen will integrate these into shear force and bending moment, no difficulty results by leaving them as they are, providing overall balance of forces is respected. (Overall stressing calculations refer loads to nodal points for local stressing anyway.)

For subsonic speeds the spacing of reference points in load calculations caters to tip effects. For supersonic speeds the aerodynamic influence coefficients are most easily evaluated where the points involved are effectively part of an infinite surface; where tip effects are involved, modification should strictly be made, but theory is not well in agreement with experiment in these regions, and in practice their need is doubtful, especially on the small area affected on the Lightning. Corrections for trailing-edge effects are in reasonable agreement with theory and were always incorporated.

Any account of loading calculations would be incomplete without mention of two other effects—namely, body lift, and the system of forces given giving pitching moment without normal force. For the former, slender body theory was used with a “cross-flow friction” correction determined empirically, with terms for intake lift based on change of stream tube momentum. This proved satisfactory.

Pitching moment coefficients at zero lift remain difficult to predict; attempts at correlating past measurements on other aircraft against factors such as the product of body volume times wing-setting angle (as suggested by slender-body theory) did not produce satisfactory results even on low-speed data. Tunnel results were suspect, being very sensitive to inclination of flow field, as tests on inverted and upright models showed. Consequently, a toleranced band has been used overall, associated with an equivalent load distribution which is not

too far removed from theory in shape, with special attention to cockpit and to all body settings. The system of forces applying at zero lift will have other effects than longitudinally—for example, wings with camber varying appreciably across the span will have modified bending moments.

## LATERAL CASES

The directional stability of aircraft at high incidence and/or high Mach number is decided by the difference between a falling fin contribution and a sustained destabilizing body effect (leaving ventral fins aside); consequently, the aeroelastic corrections to the fin term can become very important, and there is some suggestion that the forebody can contribute when derivatives corrected for aeroelasticity are fed into dynamic calculations. These calculations have to be made at many flight conditions, for many pilot actions (using real time simulation), and the search for critical cases—and critical instants—therein is a tedious one.

### SPANWISE BLOCK LOADING DISTRIBUTION FOR A MACH No OF 1.2 AND VARYING DYNAMIC PRESSURE

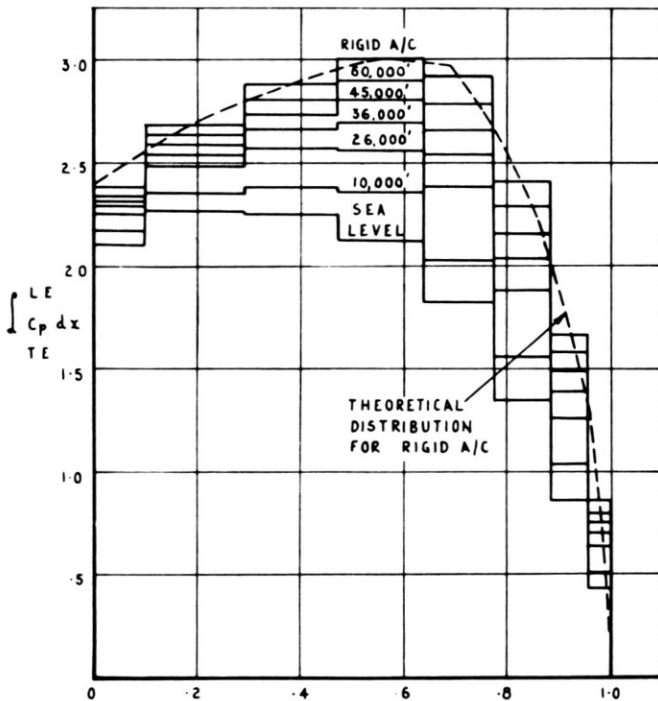


Fig. 16.

## REQUIREMENTS OF MORE RECENT PROJECTS

Performance requirements are progressively becoming more exacting and in order to minimize weight, the more recent a project the more it must enjoy the full flutter and aeroelastic treatment from the start.

Where flight through turbulent air provides a prime requirement, care will have to be taken to avoid those frequencies which have been shown to be coincident with the natural resonances of the human body. This necessitates the computation of normal modes in the early layout stages, which is not a great hardship as weight calculations are now virtually a first stressing from which the stiffnesses can be deduced, and for which mass distributions are needed in any case. Departures from this idealized case will have to be carefully watched during the later stages of design in view of previous experience on cutouts, especially now that undercarriage retraction into the fuselage and fuselage engine installations are so common.

To give guidance to the project engineer in choosing crew and equipment positions or to increase fatigue life, the response to a spectrum of turbulence can

### LOW FREQUENCY SPECTRA FOR FIN ROOT BENDING MOMENT

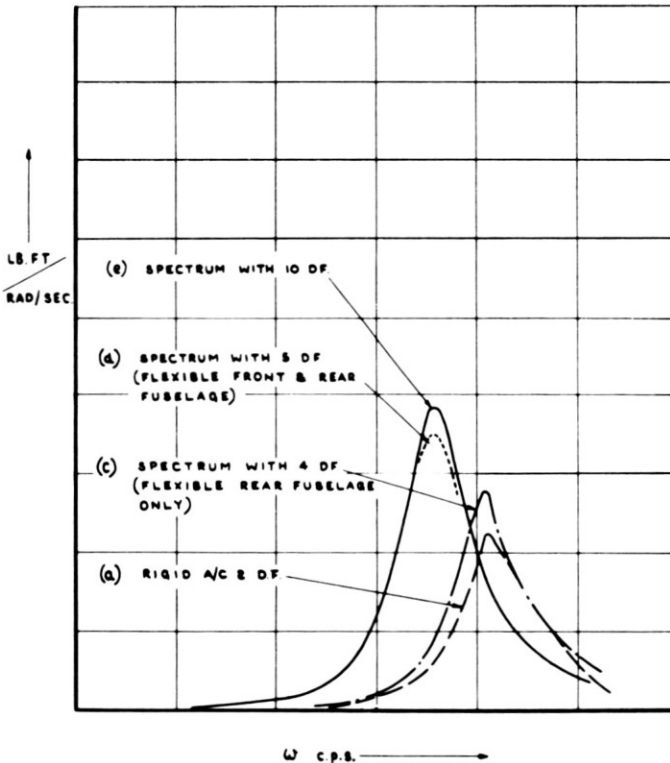


Fig. 17.



be evaluated (Fig. 17). For example, where a tailplane of large area provides good damping in body freedom modes, but has itself lower natural frequencies—would a smaller and stiffer tailplane be an advantage? The size of the fin is analogous in lateral modes.

The response calculations (for the configuration finally chosen) give the vibration levels which should be specified to equipment manufacturers, together with the frequency ranges of greatest interest to the designers of antivibration mountings. Effects of overload stores can be accommodated, including where necessary the elasticity of pylon mountings.

By far the best method of clearing flutter and other aeroelastic problems is to insure in the basic design that they cannot occur. One method of doing this becomes possible with ramjet powered aircraft, where the propulsion can be integrated with the structure. On the wing, for example, the noncircular cells so formed provide intake ducts and give a structure so deep that its frequencies are too high for flutter to occur. The drag is equivalent to that of a thin wing by virtue of the airflow through the ducts. If a triangulated Warren truss is formed, the effects of temperature variation are minimized.

Overall design considerations have never enabled English Electric to explore the beneficial effects claimed for forward slung pods in the flight stage, but the favorable dynamic magnifying of mass balance by mode shape has been utilized. In two applications the stiffness of a root rib has emerged as a primary variable. Such a solution emerges naturally from the elastic network method, but could not have been treated easily by semirigid methods.

## NOISE

Fatigue trouble due to noise has been avoided on standard aircraft at the layout stage, although several versions of the Canberra fitted experimentally with higher thrust engines exhibited cracking on the rivet lines of the tailplane, and rocket engines have caused fuselage damage.

During engine tests the skins of the Lightning intake ducts showed evidence of cracking, and although test-house resonance was believed to be responsible some local insurance stiffening was incorporated. In flight no difficulty has occurred. No panel flutter has been experienced.

## EFFECTS OF HIGH TEMPERATURE

Effects of high temperature have been considered in the Lightning for structural and equipment integrity, and empirical extension of creep laws to aging now permits the strength at elevated temperature to be assessed after heat cycling around repeated design sortie profiles. Temperature calculations have been in good agreement with measured values, except where local hot spots occur around jet pipes. Consequently, the modification of element stiffnesses and effects of their expansion can be incorporated into the stressing calculations and the deformation and load-distribution patterns defined for aeroelastic work.

As the wings encountered have still been far from solid, no loss of torsional stiffness due to self-equilibrating thermal-stress patterns acting on finite displacements has been encountered.

Power controls and the adequate provision of computing and flight-analysis facilities have changed the burden placed on the aeroelastic group as the speed and Mach number regime of modern aircraft has widened: the solution of the actual flutter equations is now the least difficult part of the task, but represents the outcome of more exacting preliminary calculations intended to advance the stage at which aeroelastic consideration can be incorporated and cleared in the design process as a whole.